J. B. Prolla, *Weierstrass-Stone, the Theorem*, Approximation & Optimization, Vol. **5**, Verlag Peter Lang, 1993, iv + 130 pp.

This short book presents a thorough study of variations, extensions, and generalizations of the classical Stone–Weierstrass theorem. It includes coverage from both a functional analytic and an algebraic point of view. However, for example, Müntz's classical theorem, which characterizes when the linear span of a sequence of monomials is dense in $C[0, 1]$, is beyond the scope of this discussion. The material is well-organized. The short chapters make the book easily readable and available not only to experts.

The book contains 13 short chapters. Chapter 1 presents the classical Stone–Weierstrass theorem for subalgebras $A$ of the continuous, real-valued functions on $X$. In Chapter 2, this is generalized to vector-valued functions. Chapter 3 discusses R. T. Jewett's proof of von Neumann's variant of the Stone–Weierstrass theorem. In Chapter 4, an extension of the Stone–Weierstrass theorem for subsets is given. The concept of multipliers associated with a nonempty subset $W$ of $C(X; E)$ plays a crucial role here. Chapter 5 is titled as "A Generalized Bernstein Theorem" and examines when a convex cone contained in $C^+(X)$ is dense in it. Chapter 6 deals with Ransford's argument for the proof of the Stone–Weierstrass theorem. By using Zorn's lemma, this avoids the use of a difficult result of Jewett that the uniform closure of a set $A$ with a property called property $V$ is a lattice. Chapter 7 discusses the Choquet–Deny theorem on the description of the uniform closure of a convex inf-lattice or a convex conic inf-lattice of functions in $C(X; \mathbb{R})$. This is applied to describe the uniform closure of the inf-lattice generated by a linear subspace of $C(X; \mathbb{R})$) which contains the constant functions. Some of these results have applications in economics. The Kakutani–Stone theorem is also proved on the description of the uniform closure of a lattice of functions in $C(X; \mathbb{R})$. This theorem is used in Chapter 8 to establish Jewett's generalization of von Neumann's theorem on the uniform closure of subsets having property $V$. Chapter 9 offers a proof of the Bishop–Machado theorem. Chapter 10 deals with the metric approximation property. All the results about approximation in $C(X; E)$ with the supremum norm can be generalized to the case where $E$ is locally convex, whose topology is determined by a family $\Gamma$ of seminorms $p$. However, when $E$ is not locally convex, many Stone–Weierstrass type questions are still unanswered. Some results that could be generalized, at least when the space $X$ is of finite covering dimension, are presented in Chapter 11. In Chapter 12, approximation processes in $C(X; E)$ are studied. Shisha–Mond- and Bohman–Korovkin-type theorems are also discussed. Chapter 13 treats the case of uniform approximation of bounded, continuous, real-valued functions defined on a space $X$ which is not assumed to be compact.

Tamás Erdélyi

R. A. DeVore and G. G. Lorentz, *Constructive Approximation*, Grundlehren der mathematischen Wissenschaften, Vol. **303**, Springer-Verlag, 1993, x + 449 pp.

Approximation theory deals with objects to approximate and approximations. This makes room for three levels: at level 1, one looks for the approximations. Books and papers on level 1 contain tables, graphs, algorithms for special functions, etc. At level 2, one studies relations between the approximated objects and their approximations: when do we have existence and unicity, rates of decrease, shapes of error functions, etc. At level 3, one investigates theoretical properties without explicit reference to any approximation process, but with roots in approximation, such as moduli of continuity and related topics, Haar systems and spaces, interpolation spaces, etc.

The present book explores mainly levels 2 and 3 (for level 1, there is only a short mention on Remez algorithm) from an elementary point of view (the advanced theory is the subject of Vol. 304 of the series, i.e., the book *Constructive Approximation: Advanced Problems*, by von Golitschek *et al.*). By elementary, it is meant that this book succeeds in giving an encyclopedic account of whatever has been achieved in real polynomial, trigonometric, and spline approximation from the nineteenth century to the present day. Much more, this impressive material is organized in 13 chapters (theorems of Weierstrass; spaces of functions; best approximation; properties of polynomials, splines, $K$-functionals and interpolation spaces; central theorems of approximation [which is indeed Chapter 7 in a set of 13]; influence of endpoints in polynomial approximation; approximation by operators; Bernstein polynomials; approximation of classes of functions and Müntz theorems; spline approximation; spline interpolation; and projections onto spline spaces) where key ideas are given with the appropriate degree of abstraction (Banach spaces, for instance), followed by concrete realizations on usual function spaces (on an interval of $\mathbb{R}$ or on the circle $\mathbb{T}$ in most cases). Proofs are given if they are not too technical or beyond the scope of the book; motivations and comments are always present. Each chapter ends with more bibliographic and historical remarks. The balance between exposition of general principles and small detail is always carefully established; for instance, the authors emphasize the importance of rearranging invariant function spaces to avoid some pathological phenomena (Chapters 2 and 9).

Obviously a reference book, it is ideal as a textbook for a solid introduction to the subject, *vaut le voyage* (*three stars*, * * *, *highest mark in the Michelin Hotels & Restaurants Guide*).

ALPHONSE MAGNUS

V. N. TEMLYAKOV, *Approximation of Periodic Functions*, Nova Science, 1993, ix + 419 pp.

The main focus of this book is the study of approximation of periodic functions of several variables. The book opens with a lengthy introduction (pp. 1–23) which contains classical results from analysis that are used in the text. Standard results on $L_p$ spaces and Fourier series and some well-known inequalities are summarized.

Chapter 1 (pp. 25–129) reviews fundamental results in the approximation of periodic functions of a single variable. Topics dealt with in this chapter include: (1) classical approximation theorem for periodic functions of a single variable, (2) Bernstein–Nikol'skii inequalities, (3) approximation of functions in certain classes, (4) width of certain classes of periodic functions, and (5) quadrature formulas and optimal recovery results. Chapter 2 (pp. 131–189) has the same structure as Chapter 1 except that it deals with functions of several variables. One of the nice features of the book is that this structure assists the reader to see more clearly how results dealing with the univariate case may be generalized to the multivariate case. One can compare Chapters 1 and 2, section by section. Chapter 3 (pp. 191–317) also employs the basic structure as set down in Chapter 1 except that it deals with functions of several variables with bounded mixed derivatives. Again, the constant structure assists the reader to make comparisons. In this chapter, the author draws heavily on his own contributions to the field. The final chapter (Chapter 4, pp. 319–390) deals with cubature formulas and optimal recovery of functions of several variables.

Overall, the book deals with an important topic which is not treated well in other books. The results presented draw heavily from journals published in the former Soviet Union. This may assist researchers from other parts of the world in developing their bibliographies. However, the typesetting and general editing of the book are disappointing. The publishers may wish to encourage the author to write a revised or second edition which is more reader-friendly.

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